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MARCH 1970 AIAA JOURNAL VOL. 8, NO. 3

Engineering Estimates for Supersonic Flutter of Curved Shell Segments

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Theoretical flutter boundaries are given for cylindrical shell segments. The problem was motivated by portions of the Saturn V booster. Donnell's cylinder equations are used in conjunction with Galerkin's method. Two static aerodynamic theories are used, one based on Ackeret theory and the other based on slender body theory. These represent short and long wavelength theories, respectively. In addition a parameter has been included that typifies the spatial pressure distribution. As this parameter is varied in a continuous manner, one can observe the effect on the panel stability of passing from the short to the long wavelength theory. The result is an upper and lower estimate for the flutter boundary, yielding a thickness requirement as a function of panel curvature and length-to-width ratio. The segments are not sensitive to the spatial pressure distribution in the short wavelength region. This may account for the relative success of Ackeret theory in predicting cylinder flutter to date.

Nomenclature

D	$= Eh^3/[12(1-\nu^2)]$
F	= Airy stress function
H	= thickness parameter, $\{[M-1]^{1/2}E/[(1-\nu^2)q]\}^{1/3}h/L$
$\tilde{H}$	= thickness parameter, $[E/(1-\nu^2)q]^{1/3}\,h/L$
h	= panel thickness
L	= length of panel
M	= Mach number
m	= axial wave number
N	= number of modes
N_x, N_θ	= stress resultants; see Eqs. (5) and (6)
$p(x,\theta,t)$	= aerodynamic load
q	= integer, also dynamic pressure
R	= radius
t	= time
V	= flow velocity
W	= width of panel
W_{eff}	= effective width of panel, W/n
w	= panel displacement in radial direction
x	= spatial coordinate, flow direction
Z	= curvature parameter, $(L/R)\,(L/h)\,(1-\nu^2)^{1/2}$
δ_{qm}	= Kronecker delta
θ	= angular coordinate
θ_0	= included angle of shell segment
λ	= eigenvalue
ρ	= fluid density
ρ_s	= panel density
Ψ	= spatial phase shift
ω	= frequency, rad/sec

Presented as Paper 68-284 at the AIAA/ASME 9th Structures, Structural Dynamics and Materials Conference, Palm Springs, Calif., April 1–3, 1968; submitted April 16, 1968; revision received October 1, 1969. This work was supported under NASA Grant NGR-23-005-166. The authors appreciate the reviewers' suggestions, which have been incorporated in the paper.

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I. Introduction

THIS theoretical study concerns the aeroelastic instability of a cylindrical shell segment. The problem was motivated by the need for design criteria for portions of the external structure of the Saturn V booster. The panel is a rectangular plate bent to a cylindrical shape and is freely supported on all four sides (Fig. 1). One might imagine the segment to be surrounded by rigid structure (Fig. 2), although the present analysis does not take the detail of the surrounding structure into account. Supersonic flow is directed parallel to the generators of the cylindrical shell segment. Primary interest is in low aspect ratio panels.

The structural side of the problem has been studied in a conventional way, with the use of Donnell's cylinder equa-

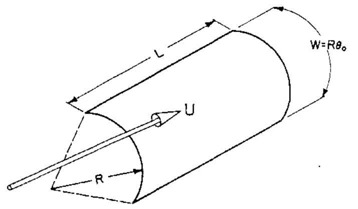

Fig. 1 Shell segment.

tions. Galerkin's method is used to find linear stability boundaries.

First attempts at the problem involved the use of unsteady potential flow aerodynamic theory. For such a panel, these relations proved to be so difficult that their use in the calculation of "coupled mode" flutter boundaries was abandoned.

On the other hand, Ackeret (linearized two-dimensional supersonic) theory has proven useful in predicting the only cylinder flutter experiments to date (Stearman, Lock, and Fung[1] and Olson and Fung[2]). This is of some interest because for the wavelengths involved in the wind-tunnel tests, it is known that Ackeret theory results in pressure expressions which are in serious error concerning the spatial distribution of pressure.

In the present study, two static aerodynamic theories are used. One is based on Ackeret theory and the other is based on slender body theory.[3] These represent short and long wavelength theories, respectively. Each of the theories is modified, however, by the inclusion of a parameter typifying the spatial pressure distribution. As this parameter is varied, the character of the pressure distribution changes from that of a short to that of a long wavelength theory in a somewhat continuous manner. This parameter gives some insight into the stability of the panel and is particularly helpful in interpreting the long wavelength theory. In the final analysis, conventional Ackeret theory and slender body theory are used to provide upper and lower estimates for the flutter boundary.

One advantage of this static aerodynamic approach is that the results can be presented in an understandable (albeit somewhat oversimplified) manner. A thickness parameter required to prevent flutter is plotted versus a length-to-width ratio. A curvature parameter enters in much the same way as in cylinder buckling work. It is felt that the design parameters used here are somewhat universal and will prove to be useful in the long run, even after more precise theories are available. Corrections to the design curves can be made as more experimental data are obtained.

The results do not include unsteady aerodynamic effects and cannot predict the single degree-of-freedom flutter typical of transonic flutter. Also, the approach used here is less accurate if used for pressurized shell segments, where frequencies are higher.

Previous work has been done on related problems. Dzygadlo[4] studied the elastic instability of an infinitely long elastic segment of an infinitely long cylinder (see Fig. 3). The stability boundaries were found for a traveling wave;

$$w(x,\theta,t) = w(\theta)\ e^{i\alpha(x-Vt)}$$

A set of integro-differential equations of motion resulted. These were solved with the aid of a Fourier series in the θ variable. Much effort was placed on a study of the effect of structural damping on the stability boundaries. For

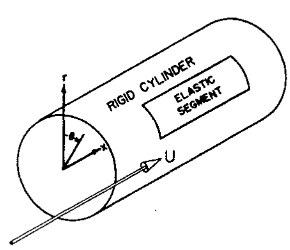

Fig. 2 Elastic shell segment imbedded in a cylinder.

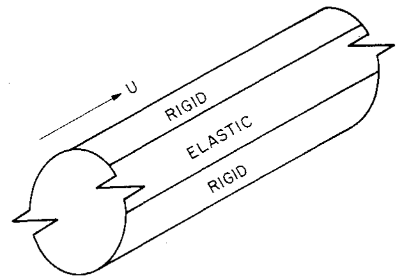

Fig. 3 Cylinder studied by Dzygadlo.

moderate amounts of damping, unexpected changes in the panel's stability resulted. The numerical results presented were not extensive. It was concluded that for small damping ratios and for fixed shell thickness and radius, the critical Mach number does not vary greatly for included angles for the segment lying between $\pi/4$ and π.

Another study of interest was by Dowell and Widnall.[3] The case considered was a finite length elastic segment in an infinitely long rigid cylindrical shell (see Fig. 4). In this case, the generalized aerodynamic forces were found for deflections of the type

$$w(x,\theta,t) = e^{i\omega t} \cos n\theta \sin(m\pi x/L)$$

Dowell made several comments about the stability of the shell segment merely by looking at the character of the generalized forces. First of all, in the low supersonic Mach number range, a single degree-of-freedom type of flutter is possible. Secondly, for long shell segments, static divergence takes place. Flutter boundaries for the "coupled-mode" type of flutter were not presented.

Another important study was done by Randall,[5] who calculated pressure distributions on stationary, three-dimensional "bumps" (such as canopies) attached to cylinders. The work dealt only with drag and the question of elastic stability of the canopy was not studied.

The present solution parallels the approach used by McElman[6] to some extent. McElman studied a curved orthotropic panel segment by using a two-mode analysis with Ackeret theory. No design curves of the type shown here were presented in McElman's work. Also, it is necessary to use many more than two modes when dealing with low aspect ratio panels.

II. Problem Statement

Consider a cylindrical shell segment as shown in Fig. 1. Supersonic flow passes over the outer surface of the segment, with flow direction parallel to the cylinder axis. The segment is of uniform thickness and of isotropic, homogenous elastic material. Conventional cylindrical coordinates x, r, θ are used. The shell segment is defined by

$$r = R, \quad 0 \leq x \leq L, \quad -\theta_0/2 \leq \theta \leq \theta_0/2$$

Deflection of the surface of the segment will be given by $w(x,\theta,t)$ measured from the mean radius of the shell. The edges of the shell will be "freely supported" as defined below. The shell may be internally pressurized. No structural damping will be included.

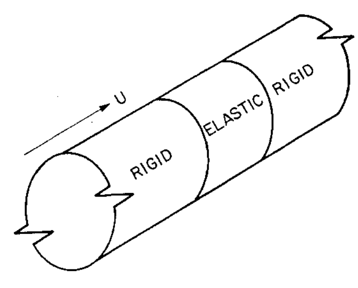

Fig. 4 Cylinder studied by Dowell.

Structural Details

The shell is thin and initially cylindrical. Radial deflections are restricted to be small

$$w(x,\theta,t)/h \ll 1$$

The in-plane motions of the shell $u(x,\theta,t)$ and $v(x,\theta,t)$ are small compared to $w(x,\theta,t)$ so that inertial effects due to in-plane motion can be neglected (Reissner's assumption). The included angle θ_0 is restricted to be $<\pi/2$ so that Donnell's shallow shell equations can be used;

$$D\nabla^4 w - \bar{N}_x \frac{\partial^2 w}{\partial x^2} - \frac{\bar{N}_\theta}{R^2}\frac{\partial^2 w}{\partial \theta^2} + \frac{1}{R}\frac{\partial^2 F}{\partial x^2} + \rho_s h \frac{\partial^2 w}{\partial t^2} + p\,(x,\theta,t) = 0 \quad (1)$$

$$\nabla^4 F - (Eh/R)\,\partial^2 w/\partial x^2 = 0 \quad (2)$$

where D is the bending rigidity of the shell, $\bar{N}_x$ and $\bar{N}_\theta$ are constants representing the components of membrane stress due to internal pressurization, and $F(x,\theta,t)$ is the stress function defined so that

$$\tilde{N}_x\,(x,\theta,t) = (1/R^2)\,\partial^2 F/\partial \theta^2 \quad (3)$$

$$\tilde{N}_\theta\,(x,\theta,t) = \partial^2 F/\partial x^2 \quad (4)$$

Note that $\tilde{N}_x$ and $\tilde{N}_\theta$ are the time dependent components of membrane stress due only to panel motion. The total membrane stresses are

$$N_x(x,\theta,t) = \bar{N}_x + \tilde{N}_x\,(x,\theta,t) \quad (5)$$

$$N_\theta(x,\theta,t) = \bar{N}_\theta + \tilde{N}_\theta(x,\theta,t) \quad (6)$$

Boundary conditions to be applied at $x = 0$, $x = L$ are

$$v = w = \partial^2 w/\partial x^2 = \partial^2 F/\partial \theta^2 = 0 \quad (7)$$

Boundary conditions at $\theta = \pm\theta_0/2$ are

$$w = \partial^2 w/\partial \theta^2 = u = \partial^2 F/\partial x^2 = 0 \quad (8)$$

The freely supported boundary conditions have been chosen primarily because they are satisfied (term by term) by the series

$$w(x,\theta,t) = e^{i\omega t}\sum_{m=1}^{N} a_m \cos\frac{n\pi\theta}{\theta_0}\,\sin\frac{m\pi x}{L} \quad (9)$$

$$\left(\begin{array}{c} 0 \le x \le \mathrm{L} \\ -\theta_0/2 \le \theta \le \theta_0/2 \end{array}\right)$$

These boundary conditions result in a plate which is dynamically "weaker" than an elastically restrained or a clamped plate. Experience has shown that an elastically restrained or clamped panel needs to be less thick to prevent flutter than the freely supported panels studied here.

At this point, the structural problem has been posed. It is necessary to find the aerodynamic pressures $p(x,\theta,t)$ generated at the panel surface.

Aerodynamic Details

Basically, the motivation for the aerodynamic expression comes from the linearized potential flow solution over an infinitely long wavy-walled cylinder. The wall deflection of such a cylinder is

$$w(x,\theta) = \epsilon \cos n\,\theta\,\sin(m\pi x/L) \quad (10)$$

which is similar to that of interest here but continued around a full cylinder. The potential solution is of the form[7]

$$p(x,\theta) = \epsilon\,(\text{const})\,\cos n\theta\,\cos(m\pi x/L + \Psi) \quad (11)$$

This expression, for short wave lengths (i.e., $L\,n \ll 2\pi R\,[M^2 - 1]^{1/2}$), reduces to

$$\text{const} = \frac{\rho U^2}{(M^2 - 1)^{1/2}}\frac{m\pi}{L} \quad (12)$$

$$\Psi = 0 \quad (13)$$

which agrees with Ackeret theory. For long wavelengths, it yields

$$\text{const} = (\rho U^2 R/n)(m\pi/L)^2 \quad (14)$$

$$\Psi = 90° \quad (15)$$

which agrees with static slender body theory.[3]

Detailed study of the full form of Eq. (11) shows that all physical cases are represented by phase angles from 0 to 90°. The magnitude of the pressure (the constant) is more difficult to typify. For our purposes, the constant will be chosen to be either the short or the long wavelength value.

Let us now return to the problem of the shell segment. Attention will be focused on two aerodynamic theories. The first will be useful for short to intermediate wavelengths. For a wall deflection of the form

$$w(x,\theta,t) = \epsilon e^{i\omega t}\cos(n\pi\theta/\theta_0)\,\sin(m\pi x/L) \quad (16)$$

the pressure will be taken as

$$p(x,\theta,t) = \epsilon e^{i\omega t}\frac{\rho U^2}{(M^2 - 1)^{1/2}}\frac{m\pi}{L}\cos\frac{n\pi\theta}{\theta_0} \times \cos\left(\frac{m\pi x}{L} + \Psi\right) \quad (17)$$

with Ψ taking values from 0 to 45°. The second pressure expression will be useful for intermediate to long wavelengths;

$$p(x,\theta,t) = \epsilon e^{i\omega t}\frac{\rho U^2 R\theta_0}{n\pi}\left(\frac{m\pi}{L}\right)^2\cos\frac{n\pi\theta}{\theta_0} \times \cos\left(\frac{m\pi x}{L} + \Psi\right) \quad (18)$$

with Ψ taking values from 45° to 90°. These two approaches will yield results in the "neighborhood" of Ackeret theory and slender body theory, respectively. The slight generalization of static slender body theory is necessary in order to detect a dynamic instability. (Static slender body theory does not lead to flutter, but to static divergence only).

There are essentially three main features to the approximations previously discussed. First of all, quasi-static theories are used—they neglect the aerodynamic forces out of phase timewise with the displacement of the panel. Secondly, the spatial pressure distribution has been chosen as a simple trigonometric function that cannot account for details of the panel geometry, e.g., the "leading edge effect." Thirdly, a spatial phase angle has been introduced to account for the uncertainty in the pressure distribution for this three-dimensional flow situation. These points will be discussed in turn.

The neglect of the out-of-phase forces is reasonable when one considers a system that flutters in a "coupled mode" type of motion (rather than a single degree-of-freedom) and if the frequency involved is low. This is because multiple degree-of-freedom instabilities are less sensitive to out-of-phase forces than single degree-of-freedom instabilities. Also, when the flutter frequencies are low, the out-of-phase forces are small. Of course, this would make the present analysis in error for axisymmetric flutter of cylinders, transonic flutter, and flutter of highly pressurized panels.

The simplification of the spatial pressure distribution simply must be done in order to get some results. In practice, the various geometries that should be covered include panels that are bounded by channels extending into the free-stream and include panels forming hat sections. This precludes any exact analysis. Theoretical studies by Platzer, Beranek, and Saunders[8] do indicate that for steady flow over an axisymmetric wavy cylinder at supersonic Mach numbers, the leading edge effect is usually small. Once one assumes a sinusoidal pressure waveform, as is done here, it then follows that the phase angle Ψ must lie between 0° and 90°.

Finally, the use of the parameter Ψ allows one to approach a difficult flow situation with some freedom to adjust the pressure distribution over a reasonable range of possibilities. The philosophy is that one is searching for an elastic instability and is not quite certain of the true aerodynamic pressure distribution. In flutter problems studied to date, the magnitude of the aerodynamic pressure has played a reasonably simple role in the stability results. On the other hand, the details of the way in which the pressure is distributed are usually much more difficult to follow. In this study, as Ψ varies from 0° to 90°, one is essentially changing the pressure distribution from that of a two-dimensional supersonic theory to that of a slender body theory.

Stability Details

Galerkin's method is used to pose the problem in matrix form. The deflections of the shell segment are

$$w(x,\theta,t) = e^{i\omega t}\cos\frac{n\pi\theta}{\theta_0}\sum_{m=1}^{N} a_m \sin\frac{m\pi x}{L} \qquad (19)$$

Note that this expression allows n half waves in the circumferential direction of the panel. If n takes a value higher than 1, then the effective length-to-width ratio of the panel increases accordingly because there are stationary nodal lines down the length of the panel.

The expression for pressure, Eq. (17), is used in conjunction with Eqs. (1) and (2) to yield the set of linear algebraic equations of motion for the short wavelength theory:

$$\sum_{m=1}^{N} a_m \left(\left\{ \left[m^2 + \left(\frac{L}{W}\right)^2_{\text{eff}} \right]^2 + \frac{12Z^2m^4}{\pi^4[m^2 + (L/W)_{\text{eff}}{}^2]^2} + \frac{N_xL^2}{\pi^2D}m^2 + \frac{N_\theta L^2}{\pi^2 D}\left(\frac{L}{W}\right)^2_{\text{eff}} - \lambda - \frac{24m\pi}{\pi^4H^3}\sin\Psi \right\} \delta_{qm} + \frac{24}{\pi^4H^3}\eta_{qm}\cos\Psi \right) = 0 \qquad (20)$$

$$(q - 1,2,\ldots N)$$

where

$$H = \left[\frac{(M^2-1)^{1/2}E}{(1-\eta^2)\,q}\right]^{1/3}\frac{h}{L}$$

$$Z = (L/R)(L/h)(1-\nu^2)^{1/2}$$

$$L/W_{\text{eff}} = Ln/R\theta_0$$

$$\lambda = \rho_s h\omega^2L^4/\pi^4D$$

$$\eta_{qm} = \begin{cases} 0 & m+q \ \text{even} \\ 4qm/(q^2-m^2) & m+q \ \text{odd} \end{cases}$$

For the long wavelength theory, Eq. (20) must be altered by multiplying the two aerodynamic terms (containing $\sin\Psi$ and $\cos\Psi$) by the factor $m(W_{\text{eff}}/L)(M^2-1)^{1/2}$. Thus, sets of linear algebraic equations are obtained. The occurrence of a negative eigenvalue λ signifies static divergence of the panel and complex λ signifies flutter.

III. Results

Stability boundaries have been calculated for the aerodynamic loading previously discussed. All results will be given for cases with zero membrane stresses $\bar{N}_x$ and $\bar{N}_\theta$.

The present study emphasizes the role played by the spatial pressure distribution. Because it is not clear how the value of Ψ affects the stability boundaries, let us choose a specific case and let Ψ vary over a wide range. (Some preliminary qualitative studies were carried out in this manner in Ref. 9).

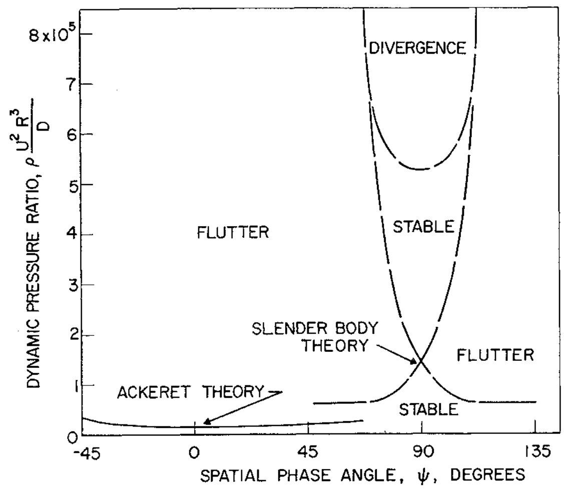

Fig. 5 Stability boundaries for a cylinder.

First of all, it should be remembered that under the present theory, there is no difference between the flutter of a shell segment and the standing wave flutter of a complete cylinder. The current theory allows no aerodynamic, elastic, or inertial coupling of adjacent panels. Hence, a complete cylinder could flutter in a mode with adjacent panels oscillating independently.

The case chosen for illustration corresponds to the cylinder studied experimentally by Olson.[2] For this case, $L/W_{\text{eff}} = 9.2$, $Z = 6,950$, and $M = 3.0$. Under the present theory, the effect of a variation in Ψ is found as is shown in Fig. 5. The dynamic pressure required to cause flutter or divergence is plotted. The short wavelength theory yields only a flutter boundary. The long wavelength theory yields both flutter and divergence boundaries.

One must decide, on the basis of figures such as Fig. 5, how to assign reasonable upper and lower bounds for the occurrence of flutter. In this case, the minimum dynamic pressure needed to cause flutter occurs at $\Psi = 30°$, whereas the maximum amount is needed for $\Psi = 90°$. It was decided to present the data in the following figures by choosing the cases $\Psi = 0$ and $\Psi = 90°$ as the limiting cases. These two limits represent Ackeret theory and steady slender body theory, which are most familiar to the average worker. In the present example, this choice causes little error. Figure 5 was obtained from a 4 mode solution.

The case at $\Psi = 90°$ is unusual in the fact that no flutter is predicted for precisely equal to 90°. (It can be seen that the flutter matrix is Hermitian and cannot have complex eigenvalues for this case.) The occurrence of flutter at nearby values of Ψ is ominous and hence the "flutter boundary" at $\Psi = 90°$ is taken as the point on the figure where the two flutter boundaries intersect (the limiting value as Ψ goes to 90°).

Stability boundaries for shell segments are given in Figs. 6–9. The results are presented using an effective length-to-width ratio L/W_{eff}, the curvature parameter Z, and the thickness parameters H and $\tilde{H}$. The plots of H vs L/W_{eff} are given as a generalization of the work of Kordes, Tuovila, and Guy and the curvature parameter Z corresponds to a parameter used in the study of cylinder buckling.[11,12] There are two subtle points about these parameters. First, the effective length-to-width ratio is not unique for a given panel. A panel with given geometric length and width may flutter in a mode with $n > 1$. Hence, a panel may have an effective length-to-width ratio which is any multiple of its geometric length-to-width ratio. Second, both parameters H and Z

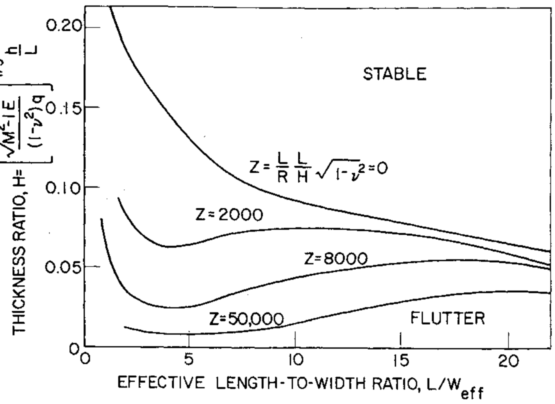

Fig. 6 Stability boundaries for a shell segment. Ackeret aerodynamic theory.

contain the thickness of the panel. Depending on the way the design curves are used, this can cause a small inconvenience (requiring a simple iteration to get a useful result when one wishes to solve for a required thickness). As presented, the design curves are most straightforwardly used when one knows the physical properties of a panel and wishes to check its stability.

It was found that for L/W_{eff} large and for small curvature, many modes were needed for convergence. Gaspars and Redd studied carefully the number of modes required for convergence on finite aspect ratio flat plates when Ackeret theory is used.[10] They found that as many as 50 modes were needed for flat plates with aspect ratios of 10 or more. When curvature is present, the results do not seem to be as sensitive to convergence problems. It was felt that 30 modes were sufficiently accurate for this study.

The solution for $\Psi = 0°$ (Ackeret theory) is given in Fig. 6. It is easily seen that curvature helps to stiffen the panel and reduce the thickness requirement. An interesting effect is obtained in the regions where H increases with increasing L/W_{eff} (positive slope). This is the case where a panel of given physical length and width will flutter in a mode with $n > 1$, giving a higher critical value of L/W_{eff}. As an example, a panel of length 10 in. and width 2 in. has a physical length-to-width ratio of 5. If $Z = 8000$ for this panel, then it might appear that a thickness ratio of 0.034 would be sufficient to prevent flutter. One must consider, however, all multiples of 5 as possible effective length-to-width ratios. One then sees that $L/W_{eff} = 20$ yields a thickness requirement of $H = 0.055$. This particular panel flutters with

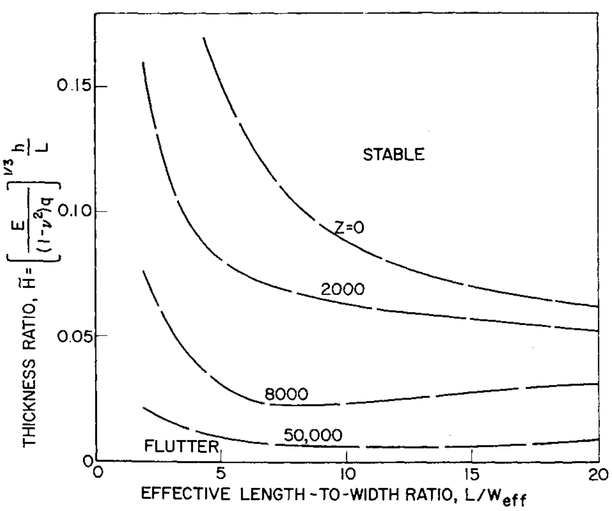

Fig. 7 Stability boundaries for shell segment. Slender body aerodynamic theory.

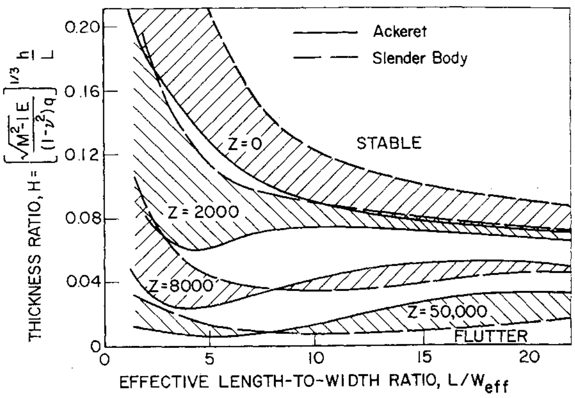

Fig. 8 Upper and lower estimates for flutter boundary. Slender body curves are presented for Mach number 3.0.

$n = 4$, i.e., it has three interior nodal lines extending down its length.

Results for $\Psi = 90°$ are given in Fig. 7. These results are similar to the $\Psi = 0°$ case, except that the ordinate is not a function of M. Again, one must observe the cases where $\bar{H}$ increases with L/W_{eff} and one must choose the multiple of the geometric length-to-width ratio which gives the largest value of $\bar{H}$.

The most useful results from a design standpoint are presented in the form shown in Fig. 8. This figure combines the results for $\Psi = 0°$ and $\Psi = 90°$ by assuming a certain Mach number and adjusting the slender body results. This figure is valid at Mach 3.0. Note that the difference between the two bounding curves is not great, particularly in certain intermediate regions of L/W_{eff}. This may be an indication why Ackeret theory gives relatively good results for the cylinder experiments discussed in Refs. 1 and 2 (see Fig. 9). As experiments are carried out, corrections can be applied to design figures such as this and some confidence in their accuracy can be established.

Other theories and experiments are shown in Fig. 9. Several of the points correspond to work for full cylindrical shells. Structurally, the major difference is that a complete, unstiffened cylinder can flutter in modes with waves travelling in the circumferential direction whereas the segment cannot. Of particular interest in Fig. 9 are the experimental points found for full cylinders by Stearman, Lock, and Fung,[1] and by Olson and Fung.[2] It is now believed that these cylinders did flutter in circumferentially travelling waves.[13] Theoretical nonlinear flutter boundaries found by Evensen and Olson give a larger thickness requirement than the current linear study. This may explain why the experimental values occurred at slightly higher values of thickness ratio than predicted by the present theory.

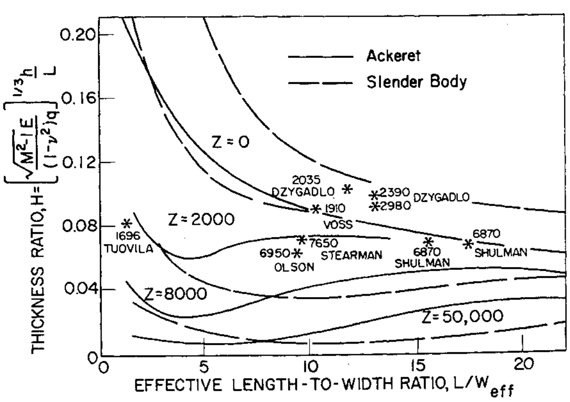

Fig. 9 Comparison with other theories and experiments. Slender body curves are presented for Mach number 3.0.

The experiments of Tuovila and Hess were carried out for a shell segment clamped all around.[14] The tests were done at Mach 1.3, which unfortunately brings transonic effects into the comparison. In transonic flow, the unsteady aerodynamic terms are of importance and these terms are missing in the present theory.

The theories of Voss and Shulman both were done for a complete cylinder with the use of Ackeret theory.[15,16] These should (and do) correspond with the present calculations and serve as a check. Their results yield thickness ratios H slightly higher than the present results. This reflects the fact that fewer modes were used. (For low curvatures and large length-to-width ratios, the thickness requirement converges from above.)

There are no unclassified experiments known to the authors which furnish the proper comparison with the theory. Such tests would be useful.

IV. Conclusions

A simplified study of the aeroelastic instability of curved panels was carried out. The assumptions made were based on theoretical solutions for flow over stationary wavy walls. A parameter was included to account for the uncertainty in the aerodynamic pressure distribution on the panel. The use of this parameter was essential in interpreting the stability of the system in the long wavelength region.

Upper and lower estimates for the flutter boundaries were presented. At a given Mach number, it was found possible to summarize most of the information for curved, unstressed panels in one design figure. Comparisons with other theories and experiments make the present analysis look reasonable, although detailed comparisons are impossible because of differences in the problems studied.

The design curves are relatively easy to use and should be suitable for rough estimates of flutter boundaries. They should be particularly helpful for wind-tunnel testing, where one is interested in both upper and lower estimates for the flutter boundary. In using the design curves, one should remember to check the stability not only at the given geometric length-to-width ratio, but also at higher multiples of this ratio.

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